Extension of a Mathematical Model using Agent-Based Simulation to Zombie Attack

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Abstract. *This paper presents an agent-based model for a mathematical model for a zombie attack. In the agent-based modeling, with the flexibility in the information and behaviors addition to environment and agents, we could obtain more realism in the simulation developed. Moreover, the information presented in the mathematical model, an extension is presented in this work, where agents search for weapons and also enter into a panic situation. There is an analogy of this model contextualized with zombies, for everyday problems, as infectious diseases or pest infestation in agriculture.*

1. Introduction

According to Ferber et al. [Ferber et al. 2003], an agent can be a real or a virtual entity, which has a perception capacity and partial representation of the environment, which can communicate with other agents, having autonomous behavior that is a consequence of its observations and its interactions with other agents. Agents can be divided in reactive and cognitive, where reactive agents only react with the environment where they inserted; and cognitive agents have a complex behavior where they can interact with other agents.

Multi-agent Systems (MAS) study the behavior of a group of distinct agents, inserted in the same environment that interact and cooperate to accomplish a particular task. The applications in MAS are more diverse, with air traffic control, business management, human-computer interaction, distributed applications, social simulation or characters in digital games.

Munz et al. [Munz et al. 2009] developed a mathematical model, presenting a zombie attack context through MATLAB software [Guide 1998]. This fanciful background reflects the traditional prey-predator environment, proposing an analogy with jobs market, where the strongest, the best-qualified and the smartest survive. However, the mathematical model is hypothetical, it does not simulate the physical contact between humans and zombies. The use of agent-based simulation allows a possibility of increasing and expanding the mathematical model with several peculiarities, making the model more realistic.

The goal of this paper is extends mathematical model using agent-based simulation and verify the results of this extension. For this, new behavior are implemented: panic situation. Therefore, this article is structured as follow: Section 2 presents the background of the work; Section 3 presents the mathematical model proposed by Munz et al.

(2009). In Section 4, we present an agent-based modeling of this scenario with all features . Section 5 presents the results obtained by this study, and, finally, Section 6 shows the conclusion of this work and future works.

2. Background

2.1. Simulation

Simulation is a method that helps in decision-making, especially in cases of medium and long-term planning or in situations that involve high cost and risk in a given context [Rebonatto 1999]. A model is a simplification of a structure or system that may be smaller, less detailed and less complex from which it observed [Gilbert and Troitzsch 2005].

Mathematical models are used to predict the involved variables values, and they are more employed in social sciences. However, the simulations have the input of data provided by the researcher and the outputs of the data are observed during the execution, presenting the behavior of the model through time [Gilbert and Troitzsch 2005].

A computational simulation is a powerful tool that can be used to model, plan and evaluate new alternatives and changes to strategies in real-world systems. Its use means building software that represent the real-world system and replicate how its works [Rebonatto 1999].

2.2. NetLogo

NetLogo is a tool that allows programming and modeling of agents for simulation of natural and social phenomena that evolve over time [Wilensky 1999]. This tool helps to visualize scenarios with graphs that illustrate the results of the simulations. NetLogo allows the modeling of simple, complex, static or dynamic systems, as well as permitting the visualization of 3D environments.

Within the tool, there are several elements called agents that can independently receive several entries according to their behavior, or save information offered by data entry. It is important to highlight that this tool is simple and intuitive, it does not require advanced programming skills to create an environment.

2.3. Mathematical Model: When zoombies attack!

Munz et al. (2009) [Munz et al. 2009] presents a mathematical model based on zombies attacks seen in digital games and TV shows, usually shown as epidemics or infestations.

In this mathematical model, initially are considered three classes: Susceptibles (S) described as healthy humans; Zombies (z) represented by humans already transformed and Removed (R) corresponds to humans killed by natural causes or in a fight with zombies.

Figure 1 illustrates the interaction of classes with the environment. Initially, susceptibles have: 1) a birth rate, represented by Π; 2) a natural death rate; 3) Can be removed (represented by $delta$). Susceptibles can be transformed into zombies through the transmission, in this case, when there is an encounter with other zombie β. Zombies can die (removed) if they are defeated in a fight (parameter α). Removed can return as Zombies,

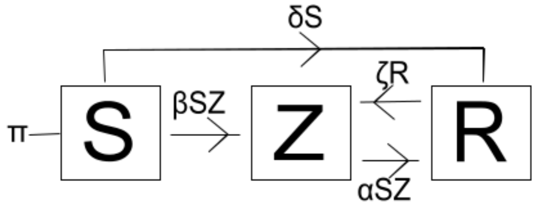

Figure 1. Basic Model [Munz et al. 2009].

represented by the parameter ζ if they are newly deceased (when a zombie comes into contact with a dead agent, killed by natural causes).

The model has three differents rules: only humans can be infected, and only zombies want to get human. These two rules exist basically to simplify the model, having interaction only between zombies and humans. The last rule is that zombies do not attack other zombies.

To model the system, the authors used Ordinary Differential Equations (ODE). [Braun and Golubitsky 1983] apud [Munz et al. 2009] explain that this approach appears in many areas of science and its applications can solve several problems, as detection in the falsification of art, diagnosis of diabetes or cell growth in tumors. ODE could be used to model population growth in a general way. Equation (1) presents an example of a differential equation that models the variation of a population of a given species over time.

$$\frac{\mathrm{d}p(t)}{\mathrm{d}t} = ap(t), \qquad a = constant. \tag{1}$$

To observe the behavior of the proposed model, the authors used MATLAB. The ODE that defines the population variation of the model were solved numerically by the Euler method for $i = 1 : n$, and Equations (2) (3) and (4) implemented in the tool.

$$s(i+1) = s(i) + dt * (-\beta * s(i) * z(i)); \tag{2}$$

$$z(i+1) = z(i) + dt * (\beta * s(i) * z(i) - \alpha * s(i) * z(i) + \zeta * r(i)); \tag{3}$$

$$r(i+1) = r(i) + dt * (\alpha * s(i) * z(i) + \delta * s(i) - \zeta * r(i)); \tag{4}$$

The Figure 2 presents a simulation in MATLAB based in basical model represented by ODE. The input variables used were α = 0.005, β = 0.0095, ζ = 0.0001 and δ = 0.0001.

The second model proposed by the authors includes a new class to the original model, called Infected (I), which is illustrated in the scheme of Figure 3. This class has a characteristic: when the susceptible is infected by the zombie, the "disease" is incubated for a period, and then the transformation into a zombie. This time for conversion would be approximately 24 hours after infection, as quoted in pop culture books on the subject and adopted by the authors of the mathematical model.

The changes that occurred in the model presented in Figure 3 consist of an infected agent could be removed before transform into a zombie because of natural death,

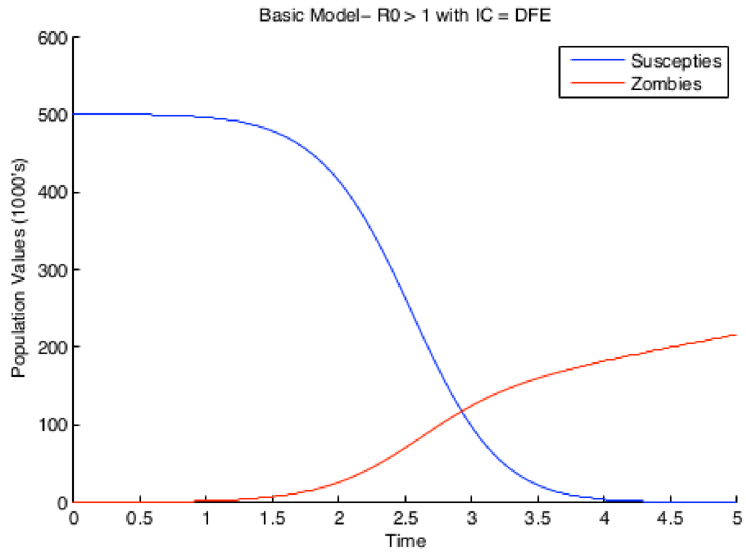

Figure 2. Basical Model [Munz et al. 2009]

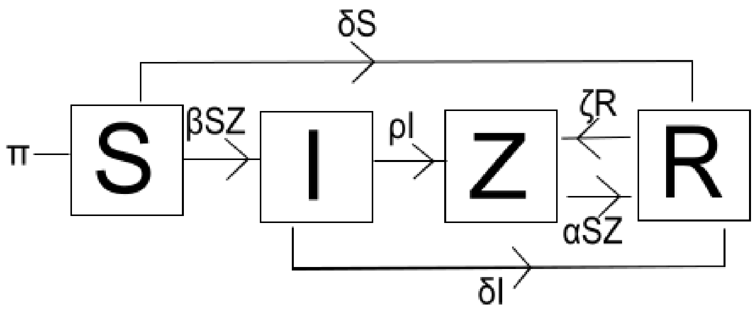

Figure 3. Model interaction with latent infection [Munz et al. 2009].

i.e., any death not directly involving zombies. Another change is the time that the transformation to zombie occurs, and this implies that even after the elimination of all susceptible, some zombies remains growing, as presented in Figure 4, this happens because the infected agents are with the virus incubated.

3. Agent-based Simulation

To develop the simulation of the mathematical model, we choose to use the perspective of agent-based simulation. In this paradigm, the mathematical model is hypothetical, since it only generates a probability of infections and deaths during the execution process. The creation of a model using agent-based simulation allows the new simulation model to be more realistic because new behaviors could be implemented as well as to add the spatial relationship of encounters between zombies and susceptible humans.

All the environment and the model to the simulations are implemented using NetLogo. The default setting defined in all runs is a grid with an open area of 40x40 patches (a toroidal model). For the first scenario, the human agent was used, having the following variables that correspond to the characteristics:

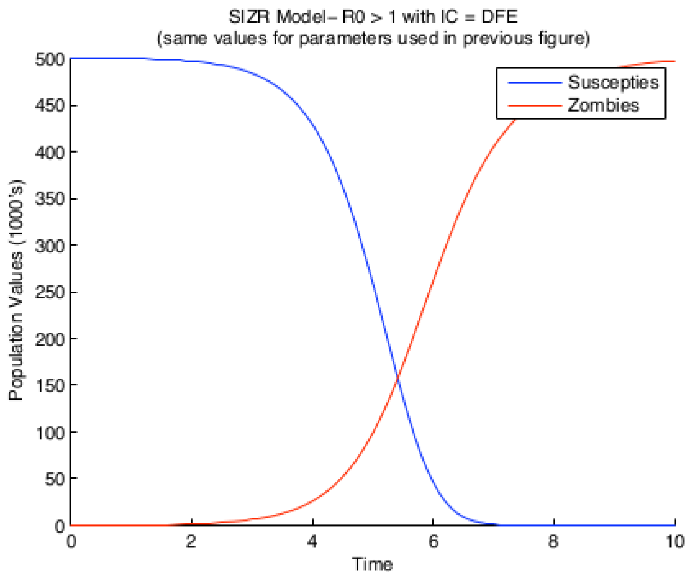

Figure 4. Basical Model [Munz et al. 2009]

- infected? : humans are not automatically transformed, the infection has an incubation period of the transformation into zombie;
- zombie? : identifies the person already changed;
- removed? : humans who are dead by natural death, or by another situation and who can still be infected and turned into zombies;
- removedTotal? : agents that no longer interact with the environment.

Unlike the simulation performed by Munz et al. (2009), which shows the population change according to time, in a simulation carried out with agents, the idea is interactions between the agents in the physical space (grid). In this way, for a human to be contaminated by a zombie, there is a need for both to located in the same patch. For that, a dispute is made, comparing the attack values of each agent: if the human wins, the zombie will totally be removed; if the zombie wins, the person can be infected, but also has a chance to be entirely removed.

In a previous work, da Costa Junior et al. [da Costa Junior et al. 2016] perform a study to provide a better interaction in the model, adding the escape behavior in susceptible agents, determining a more realistic performance with humans in danger. In order to balance the forces and give an advantage to people, there is the possibility to use weapons, increasing the attack power of the susceptibles, thus representing the adaptability and ingenuity of humans.

In the results, the authors present several simulated environments and improvements for this mathematical model, mainly based on the spatial relationship of the individuals, a fact that makes the model more realistic. They also have a relation to the number of agents - the larger the number of agents, the smaller the number of ticks (time) needed to finish the simulation.

Now, in this expansion, we have implemented a new behavior: panic. In the real world, this behavior could paralyze humans in specific situations, with no ability to move.

To perform this behavior in the simulation, it was initially set up at a rate of 10 percent of the population receiving this behavior, and these have 50 percent of chance to stay paralyzed in the current tick. Another extension of the previous work was the search for weapons. This action could represent any other resource that will bring benefit to the survival of humans, for future models. This search can configure by changing the size of the human's viewing radius, via the interface.

Figure 5 demonstrates the modeling interface in Netlogo with all the configuration options: buttons for changing the initial population, forces of attack of the susceptible and zombies, rates related to deaths by natural causes and birth rates. It also presents the options to change the scenarios with or no escape and if the susceptible agents have weapons. Still, in the interface, the grid is shown, which is where the agents interact. In the right side, we have the graph that indicates the evolution of the simulation and monitors with the corresponding quantity of each class of agent proposed in the model.

Within the simulation, each agent receives a color for a better identification in the "World". Healthy humans get blue. When a healthy human is attacked by a zombie and loss in the contest he is infected, thus receiving the color green, or if the damage is huge it can be removed entirely, the agents with this characteristic receive the white, zombies when they lose the Dispute for susceptible humans are also completely removed making it the white color. Those infected after an incubation period become zombies, turning red. Still, in this environment, we have the yellow boxes that represent the arms, and when susceptible humans find these resources, they receive like identifier black clothes with a red and red cap but keeping in the blue color the body.

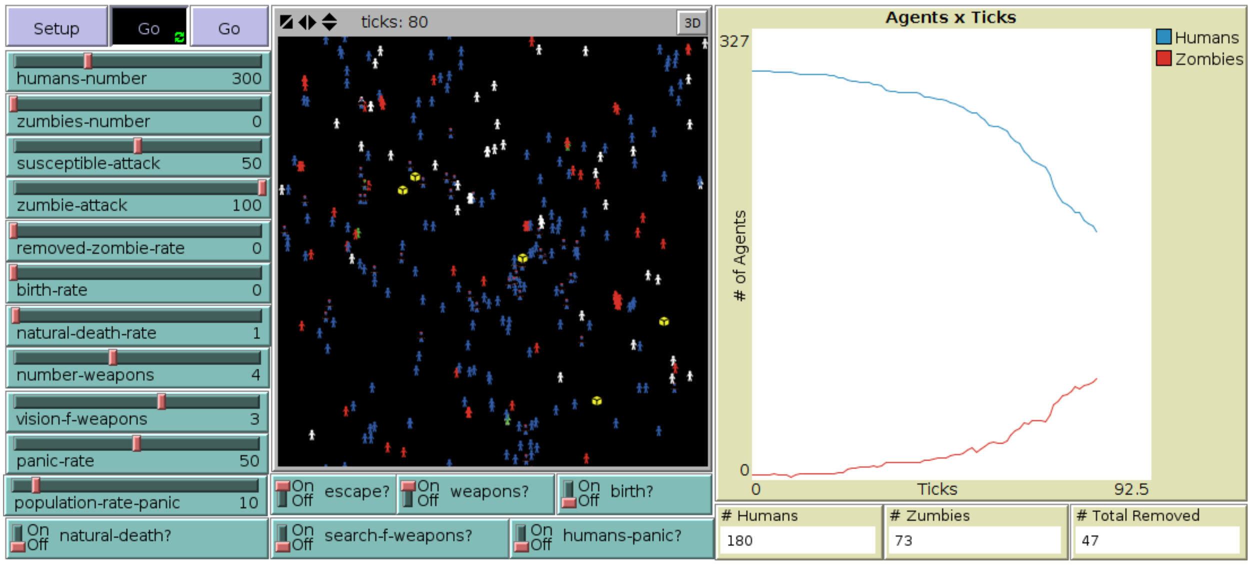

Figure 5. Model's Interface from *NetLogo*.

4. Results

We have simulated five scenarios:

- In the first scenario, agents walk randomly through the environment without any particular behavior to escape;

- In the second scenario, the susceptible agents have the ability to avoid when they perceive a zombie is nearby.
- In the third scenario, susceptible humans maintains escape behavior, and they may have weapons to increase their power;
- In the fourth scenario, they retain the ability to get away and to possess arms, and the human could search weapons;
- The fifth and final scenario is based on the situation of panic that eventually occurs with people, in which they paralyzed without reaction capacity, with all the behaviors of the fourth scene.

Mostly, in scenarios with weapons and escape ability, behavior simulation showed convergences to the basic model proposed by Munz et al. (2009), where we could compare Figure 2 with the simulation result in NetLogo, shown in Figure 6.

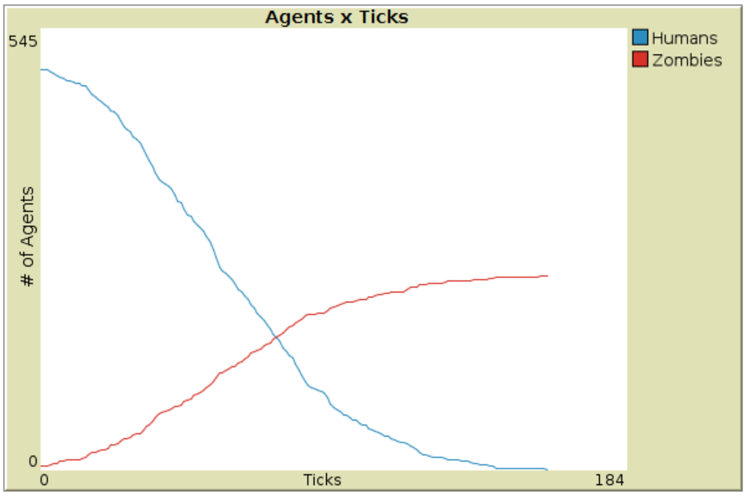

Figure 6. The behavior of agents in a simulation with weapons and escape.

However, when the weapon search situation is activated in the simulation, a new behavior occurs (emerge): susceptible humans do a circle around the weapons, as can be seen in Figure 7 (where humans are around Of the yellow boxes).

This approach is justified by the fact that susceptible humans have a gain of power when they have access to the weapons, and they could have an attack power similar to the zombies. In this way, they stay around this resource. However, when they acquire this behavior, the escape becomes more complicated, because there are many humans in a small space. Another aspect is the spatial location because the susceptible humans do not have the power to kill zombies that are not in the same field of vision.

For all five scenarios, we have performed 100 executions for the quantities of 100 to 500 agents. The averages of the results are shown in Table 1. It is possible to notice that the higher the number of agents, the smaller the number of ticks needed to complete the simulation. This conduct is explained by the little space that agents have to perform an efficient escape when this option is available in the simulation.

In Table 2, it is also possible to observe that the relationship between zombies, humans and removed, where the average is quite similar, despite behavioral changes. In fact, the significant difference is the resistance time of humans, the scenario where the

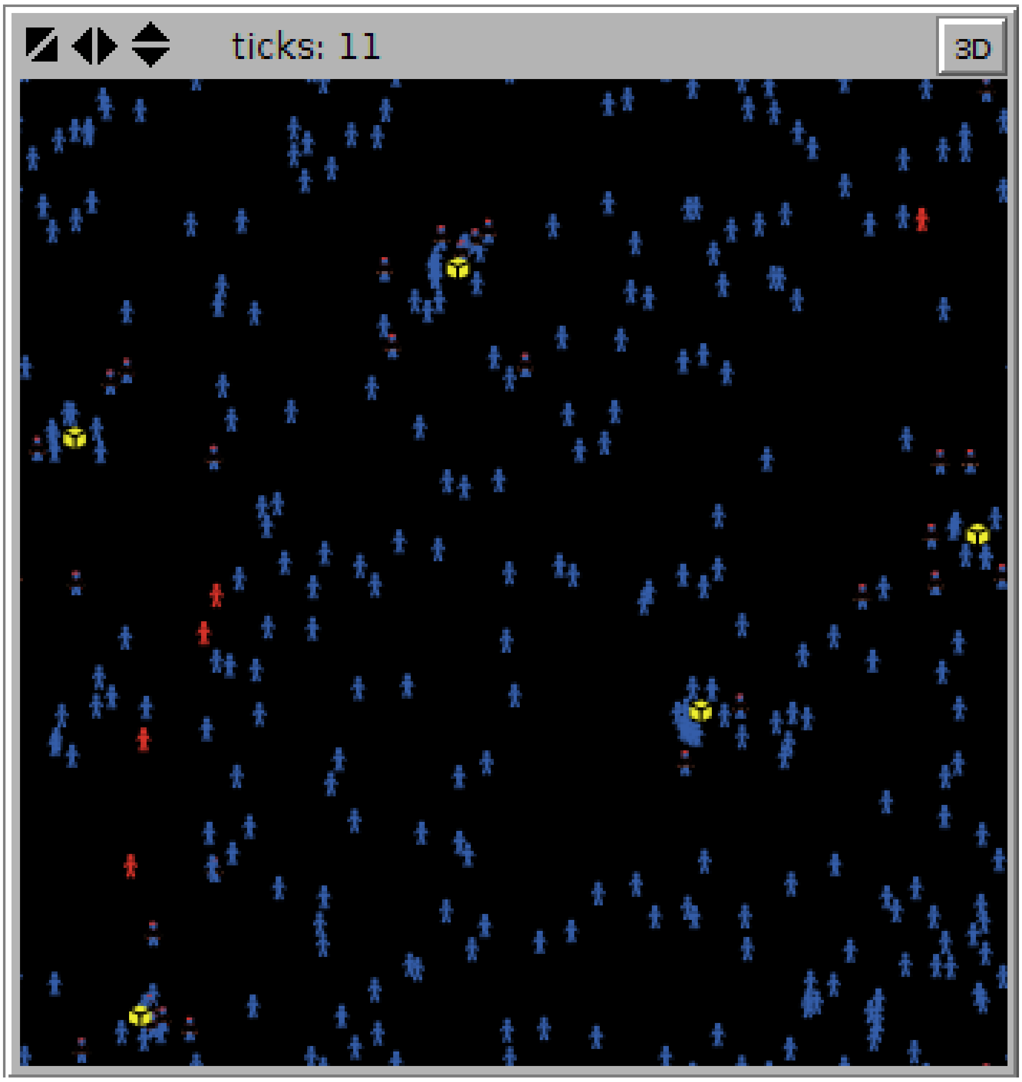

Figure 7. The behavior of humans searching for weapons.

escape is the only behavior has practically double the ticks of the situation with panic behavior. There are two conditions to conclude the simulation: the maximum number of ticks (10,000 ticks), or when there are no more human.

5. Related Works

[Wilensky and Rand 2015] presents an agent-based model that shows the propagation of a sickness in many conditions and different environments. The parameters model has an variant with the options: mobile, network and environmental. Each option influence in comunication and relationship between agents, thus generating different times of disease spread. In [Wilensky 1997], the model simulates the spread of the human immunodeficiency virus (HIV) infection through sexual transmission. In that work, authors presents four aspects that the user can change for analysis: tendency of the population to maintain abstinence, the average time a couple lives together, the population's tendency to use condoms and the population's tendency to perform HIV tests.

Table 1. Number of Ticks in simulations in NetLogo.

	Scenarios				
# Humans	Without Escape	With Escape	With Weapon	With Search	With Panic
100	3608,39	4045,89	3596,43	3202,75	2523,82
200	1286,69	1934,36	1173,73	1076,47	861,65
300	377,16	662,08	534,55	733,1	133,8
400	363,96	407,33	212,9	217,33	212,9
500	59,23	159,13	201,17	102,13	87,46

Table 2. Percentage of agents at the end of the simulation

	Number of Agents				
Escape	100	200	300	400	500
Zombies Remaining	37,82%	47,80%	48,29%	48,01%	46,94%
Living humans	37,40%	12,68%	4,91%	0,00%	0,00%
Total Removed	24,78%	39,53%	51,44%	52,00%	53,06%
Escape + Weapons + Panic					
Zombies Remaining	33,91%	41,46%	46,53%	46,72%	46,70%
Living humans	39,18%	18,64%	4,93%	0,99%	0,00%
Total Removed	26,93%	39,91%	48,53%	52,30%	53,30%

6. Conclusions and Future Works

Analyzing the obtained data from simulation, we observe the best results when compared to the mathematical modeling. The mathematical model only generates probability and run simulation. The agent-based model has a more realistic working with a spatial relationship of human and zombies, as well as implementing new movements, as an escape, inclusion of resources that can be obtained by people like weapons and other behavioral characteristics - panic. Besides this model is more realistic, there is an analogy of this model contextualized with zombies, for everyday problems, as infectious diseases or pest infestation in agriculture.

As future works, new cognitive behaviors can be idealized in susceptible humans, as an emotional link to nearest agents. We believe that the more cognition the agents have, the greater their capability to survive.

References

Braun, M. and Golubitsky, M. (1983). *Differential equations and their applications*, volume 4. Springer.

da Costa Junior, M. J. Z., Rodrigues, B. C., and Adamatti, D. F. (2016). Uma extensão para um modelo matemático utilizando simulação baseada em agentes. *Anales del IV Seminario Argentina-Brasil de Tecnologías de la Información y la Comunicación*, pages 300–308.

Ferber, J., Gutknecht, O., and Michel, F. (2003). From agents to organizations: an organizational view of multi-agent systems. In *International Workshop on Agent-Oriented Software Engineering*, pages 214–230. Springer.

Gilbert, N. and Troitzsch, K. (2005). *Simulation for the social scientist*. McGraw-Hill Education (UK).

Guide, M. U. (1998). The mathworks. *Inc., Natick, MA*, 5:333.

Munz, P., Hudea, I., Imad, J., and Smith, R. J. (2009). When zombies attack!: mathematical modelling of an outbreak of zombie infection. *Infectious Disease Modelling Research Progress*, 4:133–150.

Rebonatto, M. T. (1999). Um estudo sobre simulação paralela. *Mestrado em Ciência da Computação (UFRGS) Instituto de Informática. Porto Alegre*.

Wilensky, U. (1997). Netlogo aids model. `http://ccl.northwestern.edu/netlogo/models/AIDS`.

Wilensky, U. (1999). Netlogo. https://ccl.northwestern.edu/netlogo/.

Wilensky, U. and Rand, W. (2015). *An introduction to agent-based modeling: modeling natural, social, and engineered complex systems with NetLogo*. MIT Press.